# EMPOWER YOUR CRITICAL AND CREATIVE THINKING SKILLS THROUGH MATHEMATICAL PROBLEM SOLVING

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# CONTENT

- Scenarios
- What is Creative and Critical Thinking:
  - Insights from Edward De Bono
  - Attributes of Creative/Critical Thinking
- Relating Events and Creative and Critical Thinking
- Inspiring Maths and MPH (Maths) Creative Questions
- Its Relevance to Creative and Critical Thinking

# Scenarios

**Strategy for Problem Solving**

**Directors' Dispute**

**Business Territories**

# SCENARIO (1)

A colleague approached me for help 28 years ago. He could not help his 11 year old child solve a maths problem which normally is not meant for Primary 6 kids.

I was provoked by such a problematic maths question.

# THE MYSTERIOUS QUESTION

Sam had 75 apples and oranges.

He sold $\frac{3}{4}$ of the apples and $\frac{1}{3}$ of the oranges.

Then he had 25 apples and oranges left.

What was the ratio of the number of apples to the number of oranges at the beginning?

# OPTIONS TO TAKE

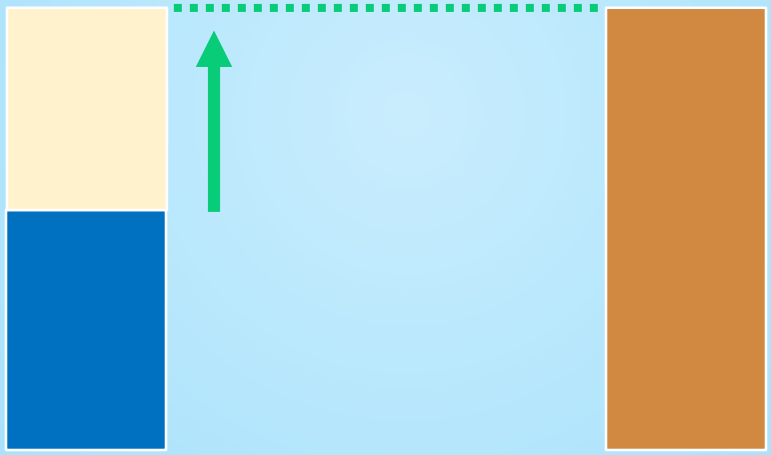
- Raise the student's ability level with the usual strategy for tackling the problem.
- Design a brand new strategy which could be understood by students at that level to tackle the higher order question.
- Abandon the problem as it is ridiculous to give such question to students of that level.

# SUMMARY OF STRATEGY

Student Ability

Question Difficulty Level

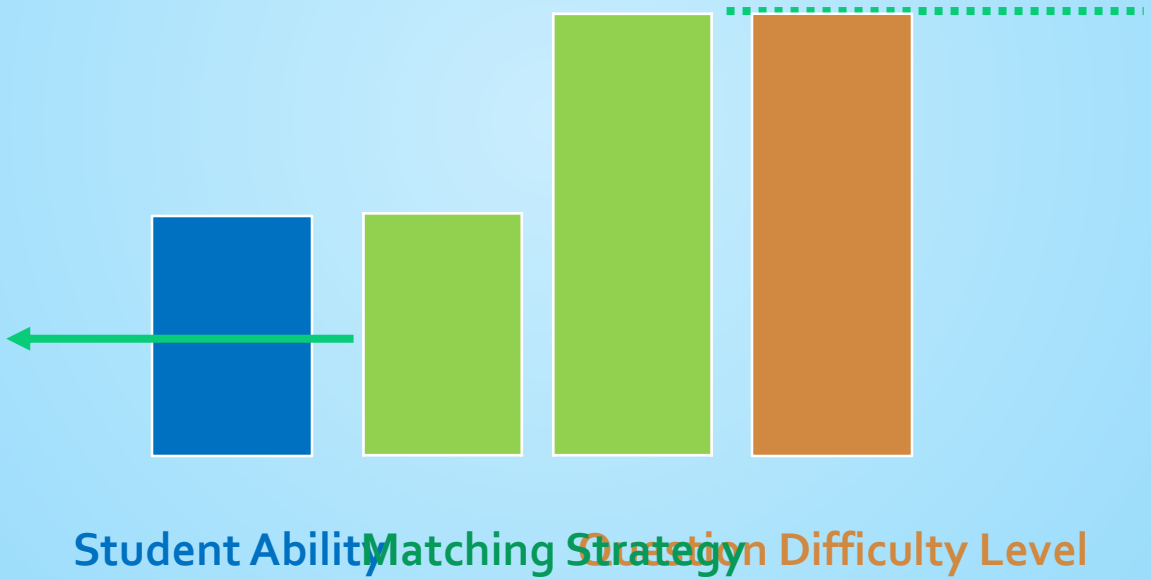
# **S1:** RAISE THE STUDENT'S LEVEL



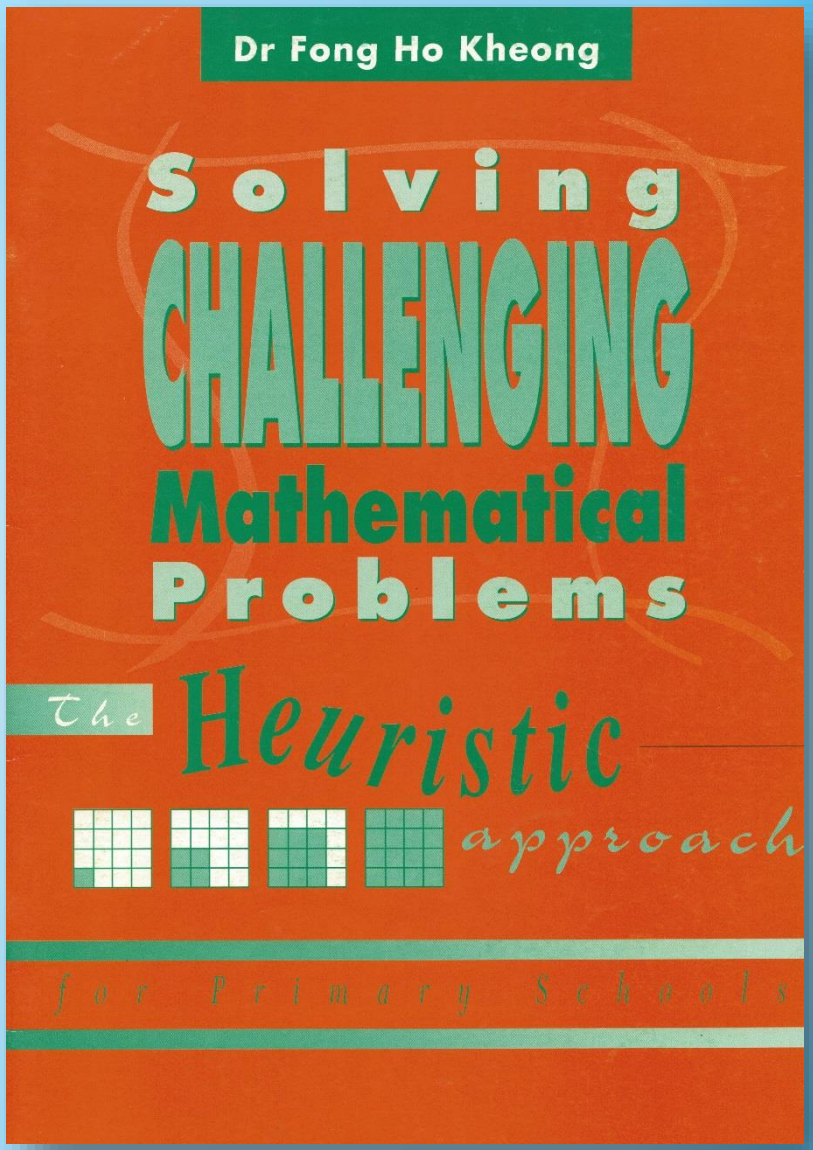
Student Ability

Question Difficulty Level

# **S2:** DESIGN A BRAND NEW STRATEGY



**First**
**Heuristic Book**
**Created by**
**Dr Fong**



# SCENARIO (2)

Dispute between two company directors.

One director started a new company and another director felt that it was a conflict of interest.

How to resolve this problematic issue?

These two directors were very good friends.

# OPTIONS TO TAKE

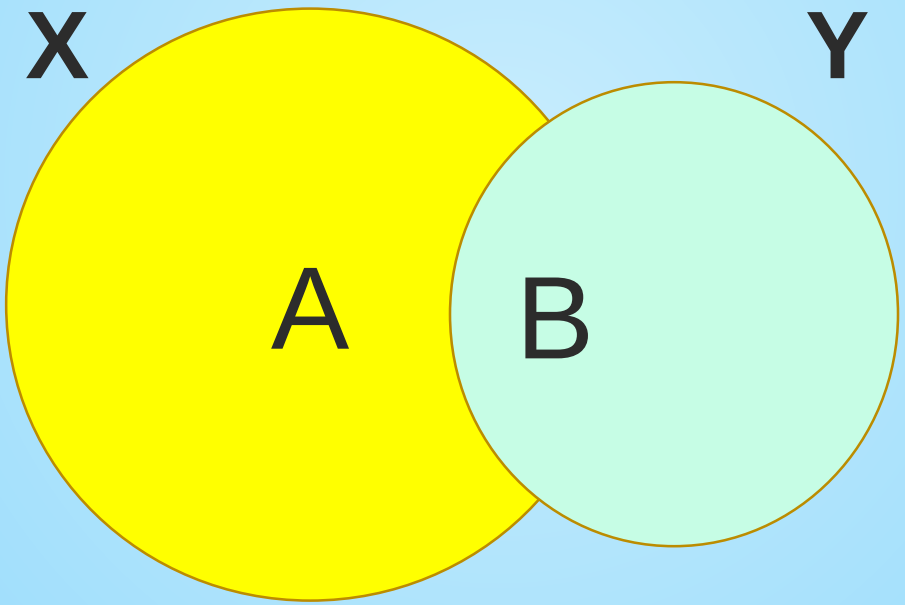
- The director who is not a shareholder of the new company is to be given some shares.
- The director who owns shares of the new company is asked to quit as a director of the current company.
- Lift the current constraints so that every one can open new companies.

# SCENARIO (3)

Company A operates a business in territory X.
Company B identifies a place to operate in territory Y but it is at the fringe of territory X.

The issue is: Should Headquarters allow Company B to operate at the identified site as the 2 companies' territories would be encroaching on each other.

# DEMARCATION OF TERRITORIES



# OPTIONS TO TAKE

- Company B is rejected because it has encroached into Company A's territory.
- Company A is given the option to expand to Territory Y so that Company B cannot enter.
- Since Company B has encroached into part of Company A's territory, Company A becomes the landlord and Company B the tenant. Arrangements can be made in terms of profit sharing.

# What is Creative & Critical Thinking?

**Insights from**

Edward de Bono

# ATTRIBUTES OF CREATIVITY

- Provocation
- Deliberate & Formal
- Exploration
- Alternatives

*(Source: Edward de Bono)*

# PROVOCATIVE ATTRIBUTES

- Jerking thinking **Out of Norm**
- **Deliberate use** of Provocations

*(Source: Edward de Bono)*

# ALTERNATIVES

- Explore different alternatives
- Remove constraints
- Use reversal method

*(Source: Edward de Bono)*

# CRAZY PROVOCATIONS?

- **Free prisoners** and set up a small business for them.
- **Close down** the **sales department**.
- Cars should have **square wheels**.

*(Source: Edward de Bono)*

Relating Events to Creative and Critical Thinking

**Provocations**

**Alternatives**

# PROVOCATIONS

**Scenario 1**: I was given a problematic question which was not meant for Year 6 students in schools.

**Scenario 2**: A director went out of his way to set up a competing business.

**Scenario 3**: Conflict of setting up business in a new territory.

# ALTERNATIVES

**Scenario 1:** 3 alternatives were suggested: student & strategy variables.

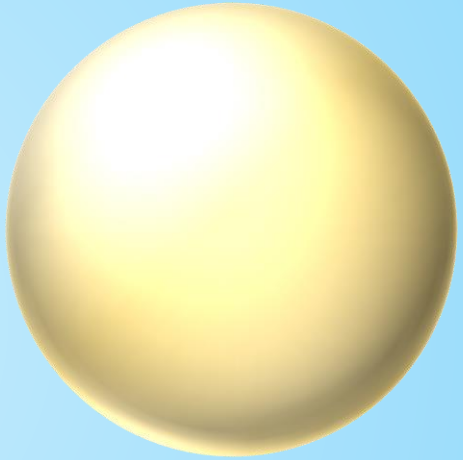
**Scenario 2**: 3 alternatives were suggested: shares given; fire director; lift constraints.

**Scenario 3**: Win-win business; landlord-tenant business model.

Students' Critical Thinking Levels

Scenario (4)

# PROVOKING QUESTION



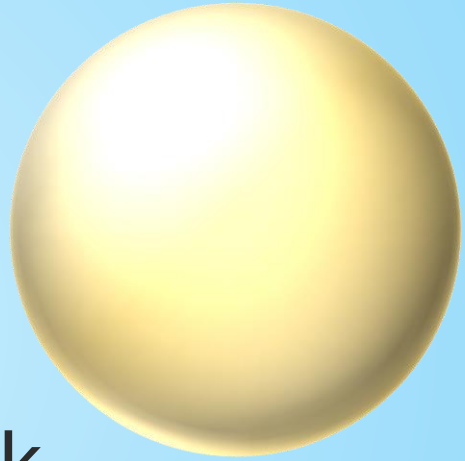
I have a glass ball.

I drop it to the floor.

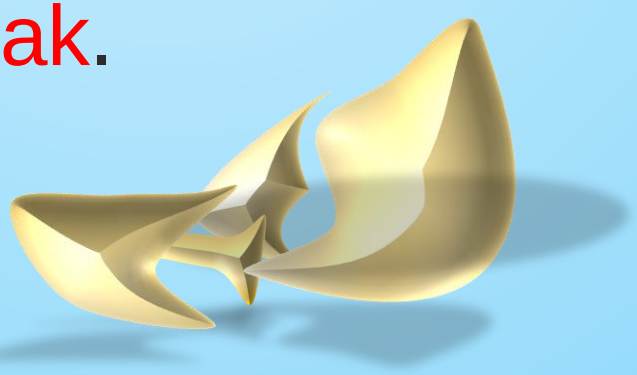
What would happen to the glass ball?

# RESPONSES: YR 1-3 STUDENTS

**S1:** It will break. If it's ice, then it wouldn't break.



**S2:** The glass ball will break. When you want to throw it, it will break.



# RESPONSES: YR 4-6 STUDENTS

**S3:** If it's on carpet, it will just roll, it will not break. But then if it's on hard surface, it will break.

**S4:** Doesn't. It depends what it lands on. It probably will break if it lands on something hard, but if it lands something soft it might not.

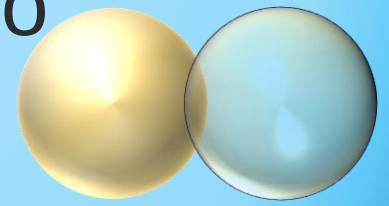
# RESPONSES: YR 4-6 STUDENTS

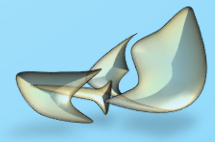
**S5:** You drop it to the floor. Is it fluid? Is it totally glass or is there some air inside? If the glass ball is thin…umm circle ball it then it will definitely break, unless you put something under it, to stop it from  falling.

# RESPONSES: YR 4-6 STUDENTS

**S5: (Continued)**

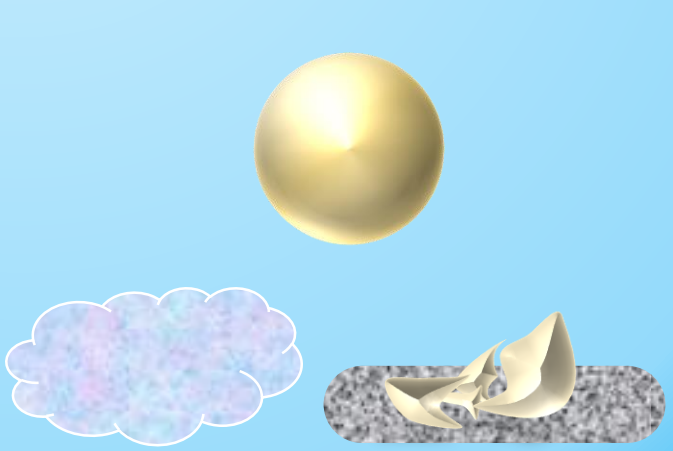
If it's filled all glass, it probably will break. If it's a thick shell, and has air, it probably wouldn't break unless you push it down or do something too hard.





# RESPONSES: YR 4-6 STUDENTS

**S6:** It will break. Depending on what the floor is? Like if it's concrete. But if it was something soft then, puffy I guess, it wouldn't break. Just bounce off, or roll off.



# RESPONSES: YR 7-9 STUDENTS

**S7:** Would it shatter? Is the ball solid? If the ball is solid, I believe it would shatter… It depends, depends on what the ball is landing on. That could be a factor. How big is the ball? I don't know. I think maybe the smaller the ball, and is solid, like a marble, I don't think it will break.

# RESPONSES: YR 7-9 STUDENTS

**S8:** It will probably break. Then it will split into lots of different sized pieces. You shouldn't drop a glass ball. And depending on the height you drop the glass ball, it's if it will break on the floor or not.

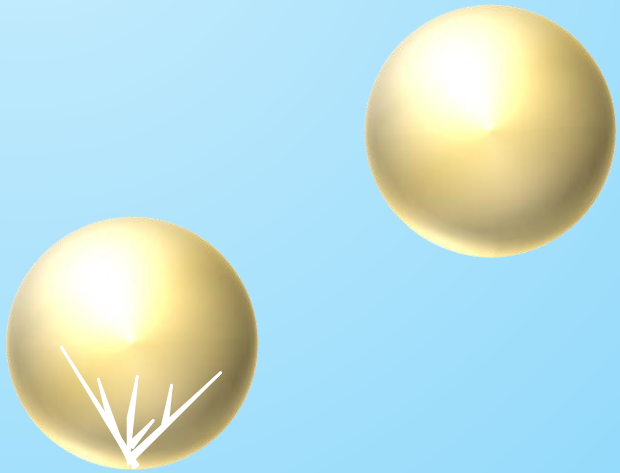
# RESPONSES: YR 7-9 STUDENTS

**S9:** It smashes. It just breaks. The ball could have fallen onto a cushion and… depending on how hard the ball hit the ground. For example, if it's a really high place, the force is more. But if it's falling from a shorter distance, it would break into not as many pieces because the force is less.

# RESPONSES: YR 7-9 STUDENTS

**S9: (Continued)**

If it's light, like when your phone screen cracks, but it doesn't really break.



# SUMMARY OF RESPONSES

**Factors:** a) Consequences of ball

b) Types of floors

c) Texture of ball

d) Size of ball

e) Height fallen

f) Impact

# FACTOR A: CONSEQUENCES OF BALL

1/ Break or do not break

2/ Shatter into pieces

3/ Size of pieces depends on height

# FACTOR B: TYPES OF FLOOR

1/ Concrete

2/ Hard, soft

3/ Carpet, cushion

# FACTOR C: TYPES OF BALL

1/ Solid

2/ Glass ball filled with air

3/ Thickness of outer surface

# FACTOR D: SIZE OF BALL

1/ Big or small

# FACTOR E: HEIGHT OF BALL FALLEN

1/ How high the ball has fallen?

# FACTOR F: IMPACT

1/ How hard it hits the floor?

# HIERARCHY OF RESPONSES

| School Year of Students | Types of Responses |
|---|---|
| 1-3 | Ball will **break**. |
| 4-6 | Break or not break **depends on** types of **floor surfaces**.<br>Depends on **texture** of glass ball. |
| 7-9 | Depends on **floor surfaces**.<br>**Size** of the ball.<br>**Height** of ball.<br>**Impact**.<br>Types of **smashed pieces**. |

Critical Thinking Other Perspectives

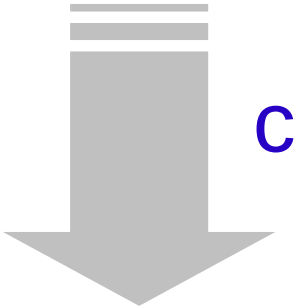
Attributes

# ATTRIBUTES OF CRITICAL THINKERS

- Raise vital questions and problems.
- Gather & assess relevant information.
- Think open-mindedly within alternative systems of thought.
- Communicate effectively with others.
- Come to well-reasoned conclusions and solutions.

# Critical Thinking Problems

# Problem Solving



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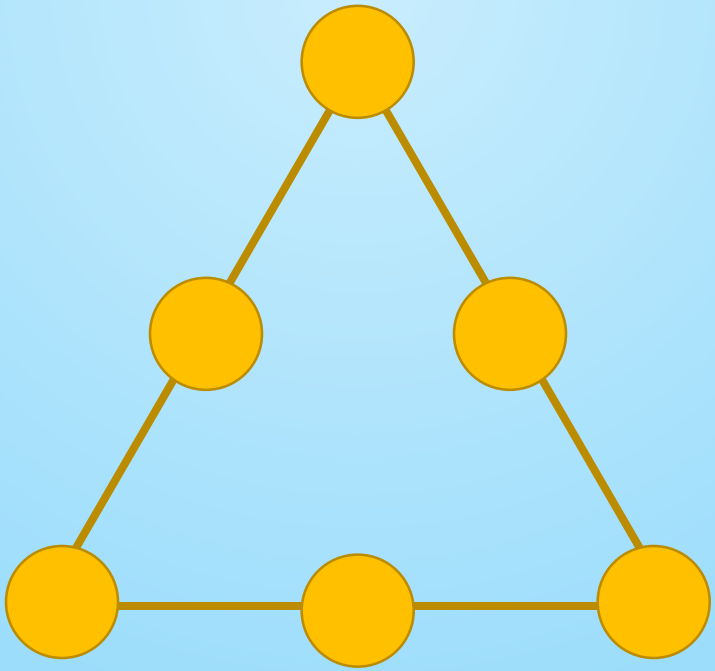
# Critical Thinking

# PROBLEM 1 (INSPIRE MATHS)

Fill in the circles with numbers 4, 5, 6, 7, 8, and 9.

Each  makes 20.



Source: Inspire Maths

# CRITICAL THINKING

**Recall Number Bonds (Gather Information)**

**Raise Vital Questions: (Alternatives)**

a) What strategy to use

b) Any short cuts

c) Position of circles

**See Connections:**

Numbers at corners appear twice

# SOLUTION

List of Number Bonds:

4-**7**-**9** → 20

**5**-**7**-8 → 20

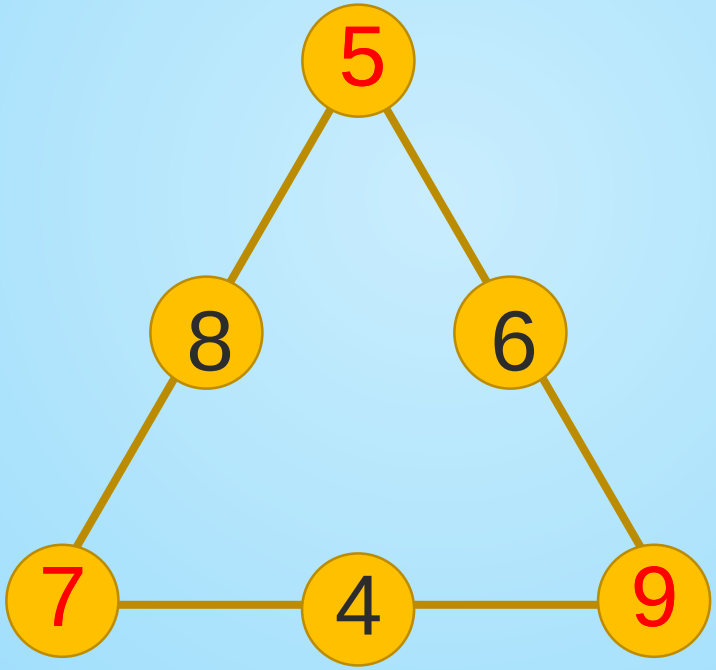
**5**-6-**9** → 20

Connections:

Numbers at corners form 2 number bonds 5, 7, 9.

Complete the rest.

# SOLUTION



# PROBLEM 2 (INSPIRE MATHS)

Jack arranged two different square pieces of paper as shown.

The length of the side of each square piece of paper is a whole number.

The total area of the shape is 89 $cm^2$.

What is the length of the side of each square piece of paper?

Source: Inspire Maths

# CRITICAL THINKING

What critical thinking skills are involved in solving this problem?

# SOLUTION

**List of whole numbers:**

1 2 3 4 5 6 7 8 9 10

**List of square numbers:**

1 4 9 16 25 36 49 64 81 100

**Raise Question.** Which 2 squares make 89?

**Side →** 5 8

# PROBLEM 3

Five numbers form a pattern when arranged in order.

Three of these numbers are shown below.

Find all possible pairs of numbers to form a pattern.


(Similar question also found in INSPIRE MATHS)

# SOLUTION

The 3 numbers form a pattern

68, 76, 84 (difference is 8)

# DISCUSSION

Most children will provide the following solutions:

| | | | | | | |
|---|---|---|---|---|---|---|
| | × | 68 | 76 | 84 | × | |
| × | × | 68 | 76 | 84 | | |
| | | 68 | 76 | 84 | × | × |

Is this exhaustive?

# DISCUSSION

What about this ?

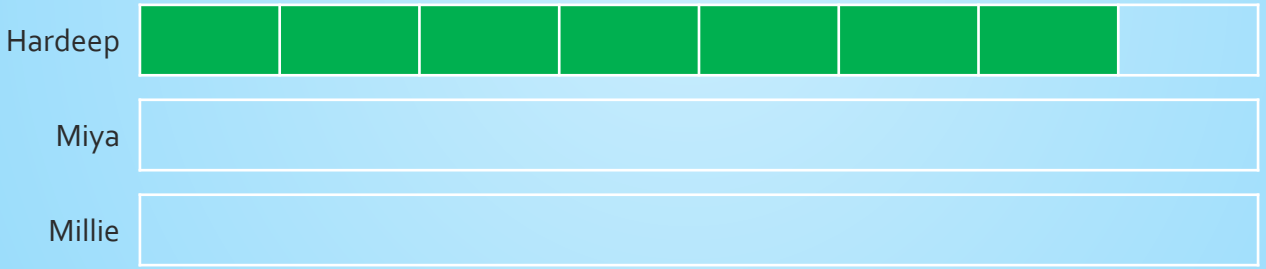
$$68 \times 76 \times 84$$

# PROBLEM 4 (INSPIRE MATHS)

Hardeep, Miya and Millie each have a fraction strip.

Hardeep's fraction is greater than Miya's and Millie's.

Miya's fraction is smaller than Millie's.




What could Miya's and Millie's fractions be?

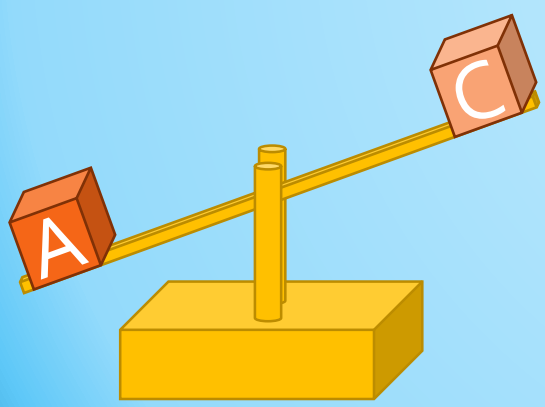
Can you think of any possible answers?

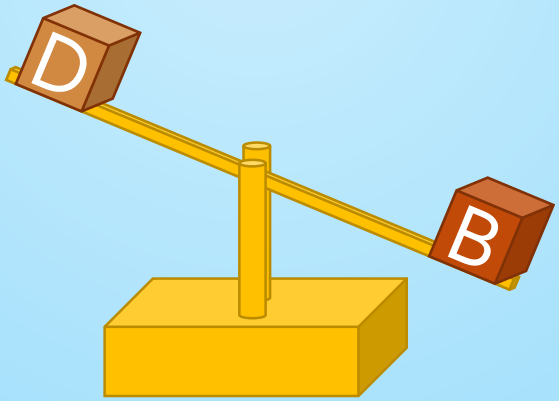
Source: Inspire Maths

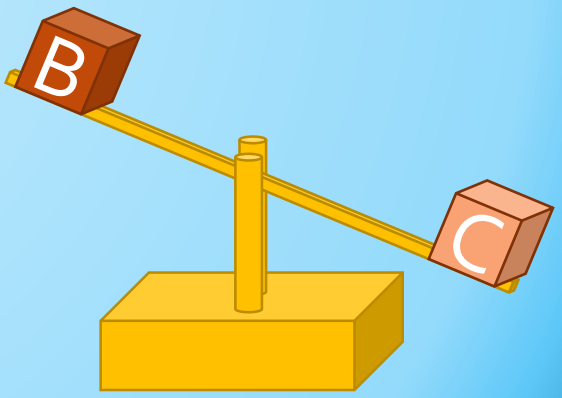
# PROBLEM 5 (INSPIRE MATHS)

Look at the pictures.

Arrange the boxes in order, from heaviest to lightest.




Source: Inspire Maths

# PROBLEM 6 (INSPIRE MATHS)

Ella arranges 10 beads in a row.

There is only one red bead.

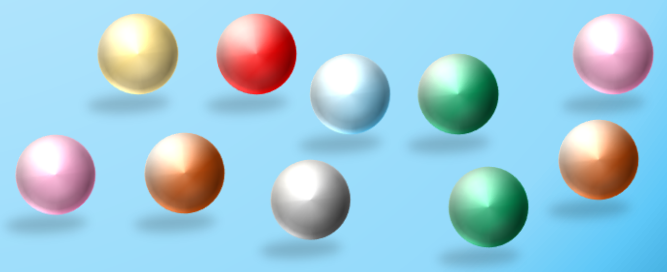
She puts it 6th from the right.

Ella counts from the left.

What position is the red bead in?

Draw a diagram or act this out with your friends.

Source: Inspire Maths



# PROBLEM 7

Peter has chickens and rabbits in his farm.

The animals have 40 legs and there are

14 animals in total.

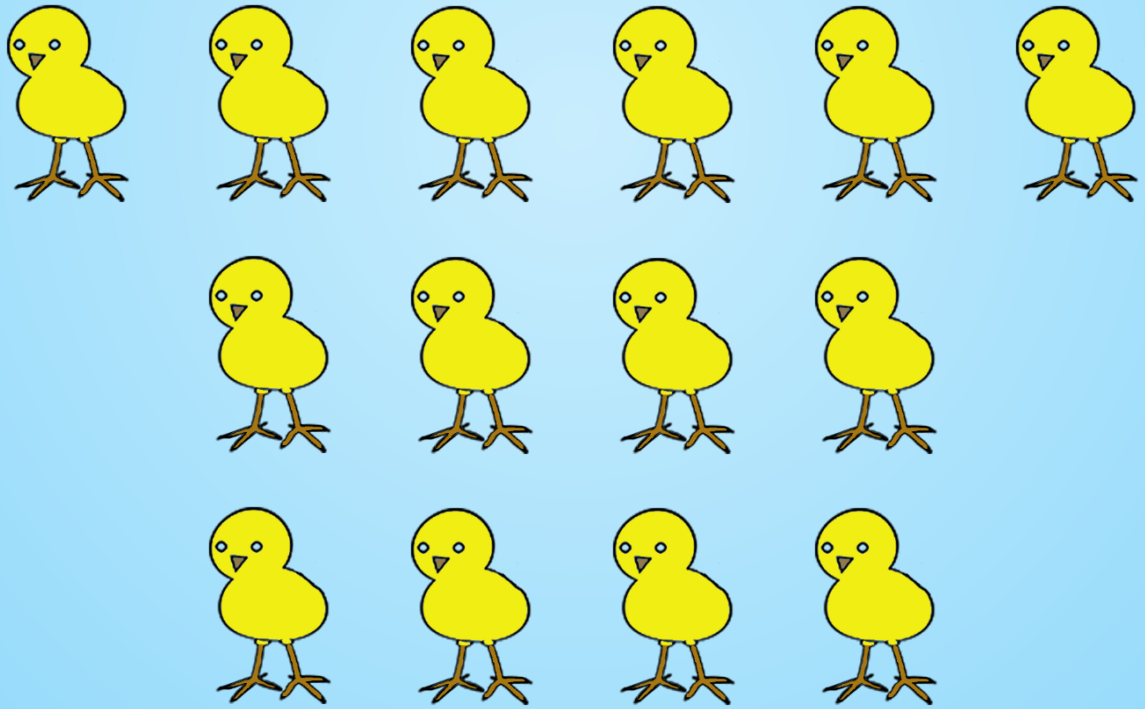
How many rabbits does Peter have?

Show at least **3 different methods** to solve this problem.

(Similar question also found in INSPIRE MATHS)

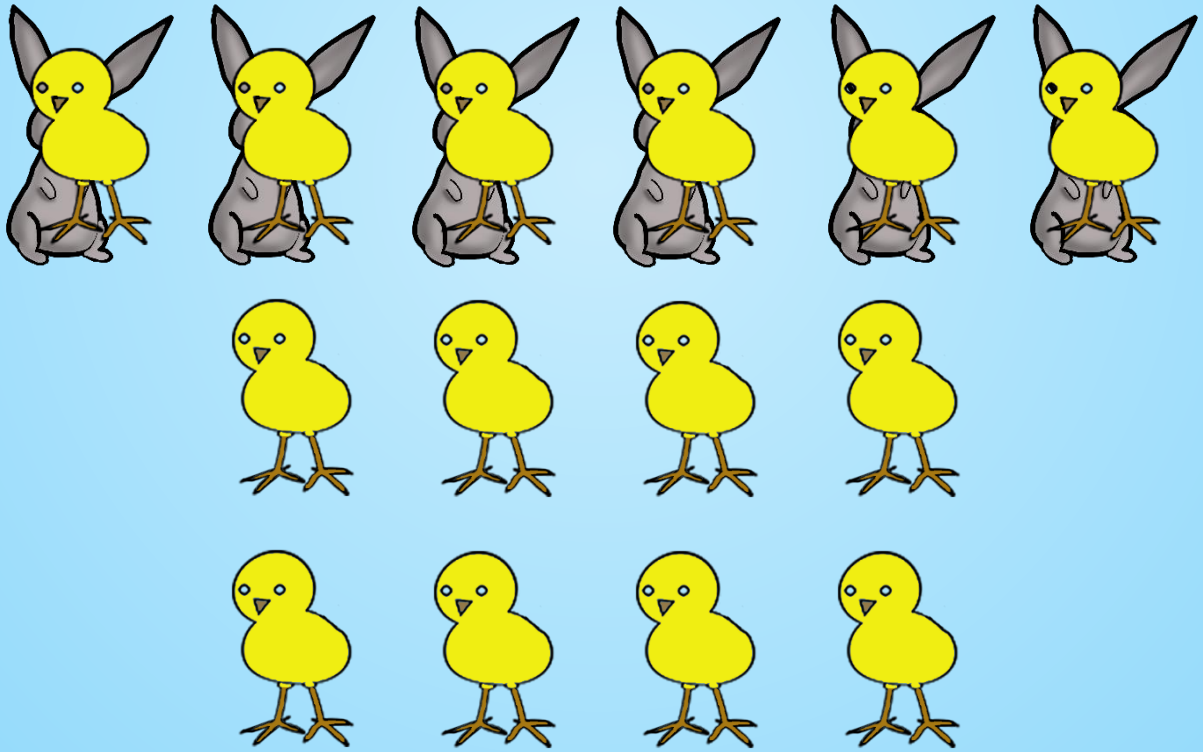
# DRAWING DIAGRAM METHOD

- All the animals are chickens with 28 legs altogether.



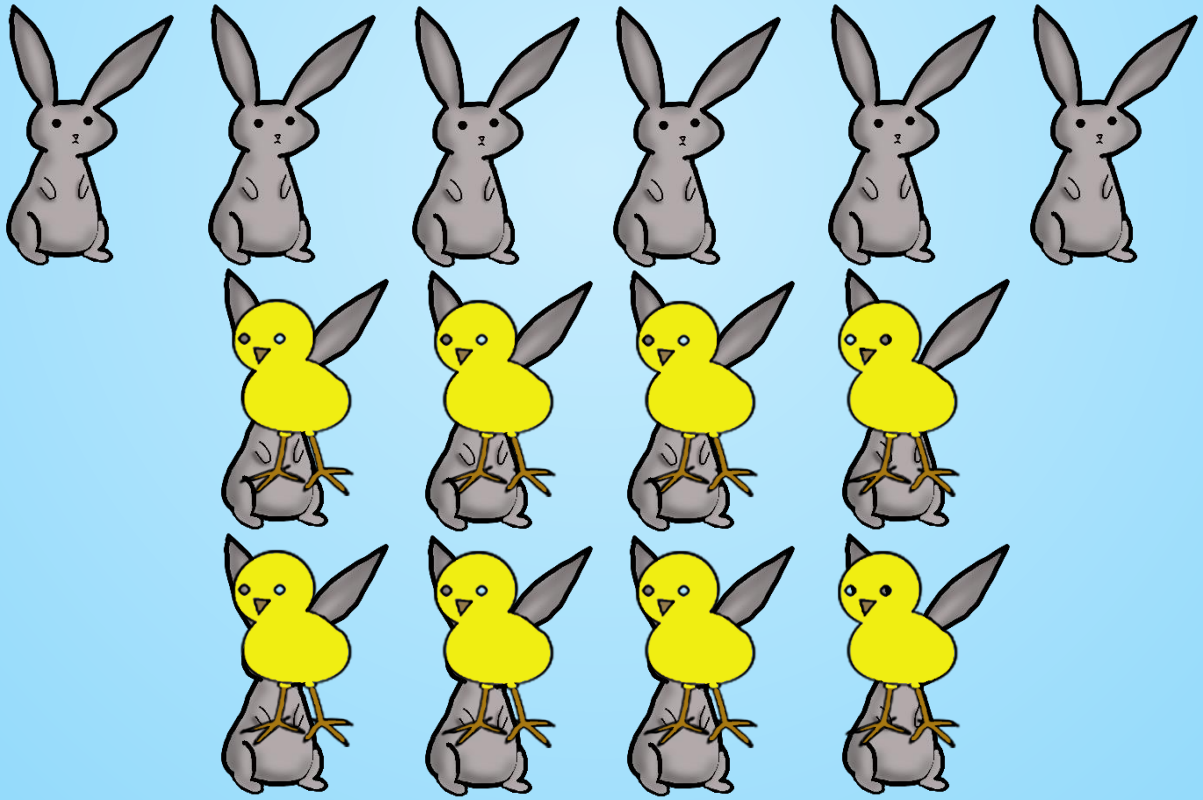
# DRAWING DIAGRAM METHOD

- There are 40 legs altogether. 6 rabbits and 8 chickens



# DRAWING DIAGRAM METHOD

- Can I assume all the animals are rabbits? How to proceed?



# WHAT IS CREATIVITY?

**Creativity** is a phenomenon whereby something new and somehow valuable is formed (Wikipedia).

**Creativity** is something original and worthwhile (Source: Sternberg).

Creative Thinking Perspectives

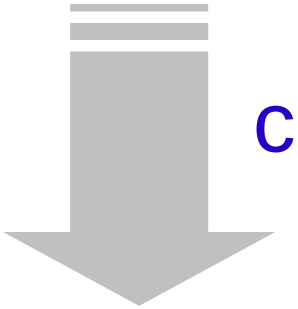
Attributes

# ATTRIBUTES OF CREATIVE THINKERS

- **Perceive the world in new ways.**
- **Openness** and **exploratory** behaviour.
- **Make connections between seemingly unrelated phenomena and**
- Find general solution.

(L Nailum)

# Problem Solving



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# Creative Thinking

# PROBLEM 1

Use the following words to write a subtraction story.

| | | |
|---|---|---|
| 20 | Sarah | at the end |
| oranges | gave away | ? |
| have | Tina | How many |
| 9 | oranges | She |

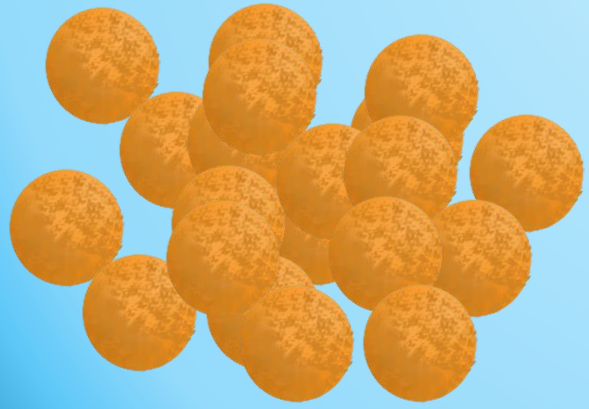
(Similar question also found in INSPIRE MATHS)

# SOLUTION

Sarah had 20 oranges.

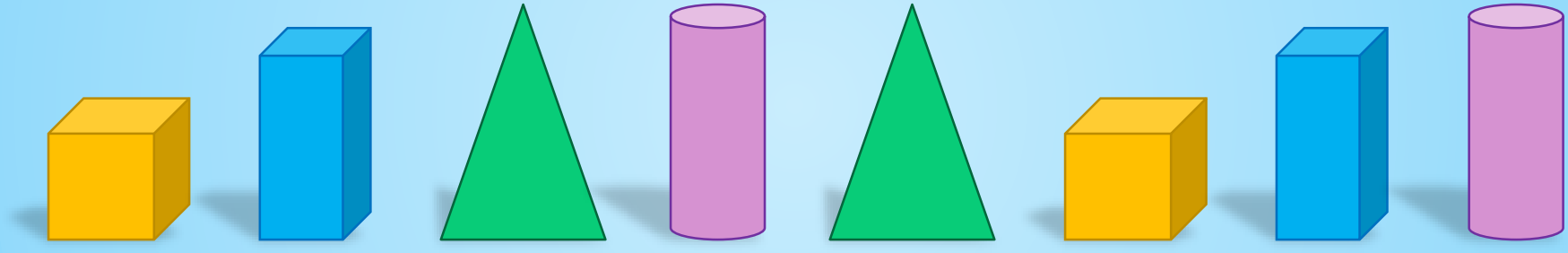
She gave away 9 oranges to Tina.

How many oranges did Sarah have at the end?



# PROBLEM 2 (INSPIRE MATHS)

You will need these objects.



Make your own patterns.

Ask a friend to show what comes next.

Source: Inspire Maths

# PROBLEM 3 (INSPIRE MATHS)

You will need these cards.


Use the cards to make number sentences.

Write down all the number sentences you make.

Source: Inspire Maths

# PROBLEM 4 (INSPIRE MATHS)

Show **two ways** to add the three numbers.

9 + 7 + 8 = ☐

Use number bonds to make tens.

Source: Inspire Maths

# PROBLEM 5 (INSPIRE MATHS)

Work in pairs.

a) Write one addition story and one subtraction story.

Use the following words to help you.

| | | |
|---|---|---|
| Jack | Miya | more than |
| Sea shells | how many | collects |

b) Write one addition story and one subtraction story.

Use the following words to help you.

Source: Inspire Maths

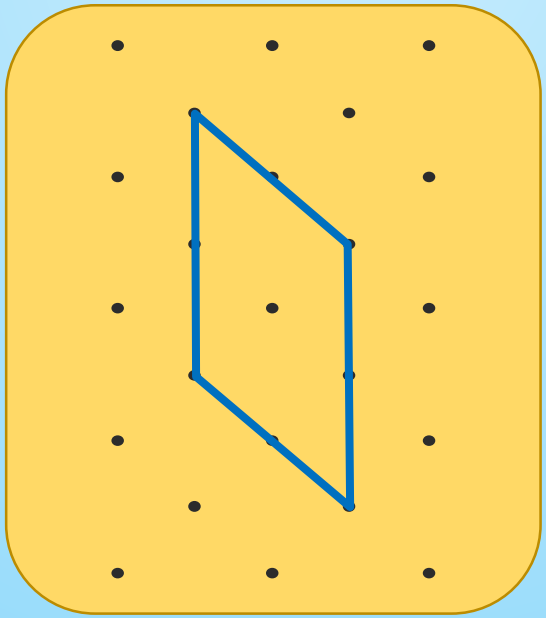
| | | |
|---|---|---|
| Bella | Tom | less than |
| Sandwiches | how many | makes |

# PROBLEM 6 (INSPIRE MATHS)

You will need some dotty paper.

Design a unit shape that can tessellate and make your own wallpaper pattern.

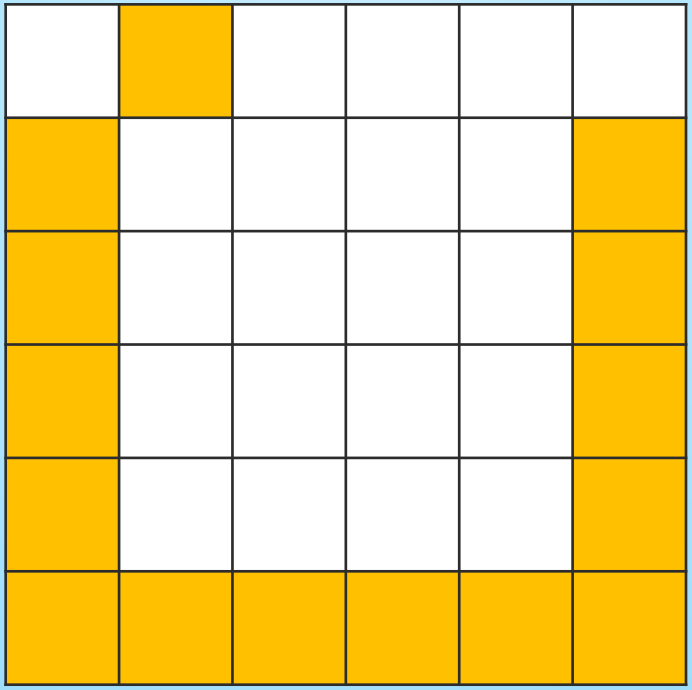
Start with a shape that can tessellate, such as the shape below.



Source: Inspire Maths

# PROBLEM 7 (INSPIRE MATHS)

By adding one or more squares show at least two or more ways to make the figure symmetrical.



Source: Inspire Maths

# PROBLEM 6 (INSPIRE MATHS)

Work in groups of four.

You will need a geoboard and some rubber bands.

Make as many different rectangles as possible on the geoboard.

Make sure that all the rectangles you make have the same perimeter.

Then write down your answers like this:

| Rectangle | Length | Width | Perimeter | Area |
|---|---|---|---|---|
| A | 3 cm | 3 cm | 12 cm | 9 $cm^2$ |
| B | 4 cm | 2 cm | 12 cm | 8 $cm^2$ |



Source: Inspire Maths

# CONCLUSION

a) Creativity requires critical thinking

b) Provocation is necessary to trigger creativity

c) Exploring alternatives is an essential attribute that leads to creativity

d) Being able to see connections is another attribute that leads to creativity

e) Mathematical Problem Solving has all the above attributes

# MINISTRY INITIATIVE (MOE SG)

**Thinking School Learning Nation:**

Focuses on developing all students into active learners with critical thinking skills, and on developing a creative and critical thinking culture within schools

**PM Goh Chok Tong (1997)**

Source: Straits Times

# END OF

# PRESENTATION

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